K23U 1992

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus and Logic

Time : 3 Hours

Max. Marks : 48

SECTION – A

Short Answer Questions. Answer **any 4**.

1. Find the value of $\int_0^{\pi/2} \cos^7 x \, dx$.
2. Find a polar equation for the circle $(x-2)^2 + y^2 = 4$.
3. What is Tautology ?
4. Write the principle of double negation.
5. Write the contrapositive of the implication "If I am in Chicago, then I am in Illinois". **(4×1=4)**

SECTION – B

Short Essay Questions. Answer **any 8**.

6. Show that $\sinh 2x = 2 \sinh x \cosh x$.
7. Evaluate $\int_0^1 x^2 \left(1-x^2\right)^{3/2} dx$.
8. Evaluate $\int \text{cosec}^5 x \, dx$.
9. Calculate $\int\int_R f(x, y)\, dA$ for $f(x, y) = 100 - 6x^2y$ and $R : 0 \le x \le 2, -1 \le y \le 1$.
10. Graph the sets of points whose polar coordinates satisfy the conditions $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$.

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11. Write the equations relating spherical coordinates to Cartesian and cylindrical co-ordinates.

12. Find the Jacobian for the polar coordinate transformation $x = r\cos\theta$, $y = r\sin\theta$.

13. Find the minimum number of intervals required to evaluate $\int_0^1 \ln(1+x)\,dx$ using Simpson's 1/3 rule with an accuracy of 10^{-6}.

14. Evaluate $\int_0^2 \frac{dx}{x^2+2x+10}$ using Simpson's rule with n = 2.

15. Find the truth set T_p of the propositional function p(x) given by "$x + 5 > 1$", defined on the $P = \{1, 2, 3,...\}$.

16. Negate the statement "All students live in dormitories". (8×2=

SECTION – C

Essay Questions. Answer **any 4**.

17. Evaluate $\int_0^{\ln 2} 4e^x \sinh x\,dx$.

18. Derive the reduction formula for $\int \sin^n x\,dx$

19. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$

20. Evaluate $\int_0^1\int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$.

21. Use truth table to show that $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

22. If m and n are natural numbers such that $m + n \geq 20$ then show that either $m \geq 10$ or $n \geq 10$.

23. Give a direct proof to the theorem "The square of an odd integer is also an odd integer". (4×4=

SECTION – D

Long Essay Questions. Answer **any 2**.

24. Obtain a reduction formula for $\int x^n e^{-x}\, dx$ and hence show that the improper integral $\int_0^\infty x^n e^{-x} dx = n!$, where n is any positive integer.

25. Using polar integration, find the area of the region R in the xy – plane enclosed by the circle $x^2 + y^2 = 4$, above the line $y = 1$, and below the line $y = \sqrt{3}x$.

26. Evaluate $\int_0^1 \frac{dx}{3+2x}$ using trapezoidal rule with $n = 2$. Compare with the exact solution. Also find the number of sub intervals required if the error is to be less than 5×10^{-4}.

27. Prove that there are infinitely many prime numbers. **(2×6=12)**